P425/2
Applied Mathematics
Paper 2
July - August, 2017
3 hours

UGANDA MUSLIM TEACHERS' ASSOCIATION
UMTA JOINT MOCK EXAMINATIONS 2017
UGANDA ADVANCED CERTIFICATE OF EDUCATION
Applied Mathematics
Paper 2
3 hours

INSTRUCTIONS:

- *Answer **all** the **eight** questions in **Section A** and any **five** from **Section B.***
- *Any additional question(s) will **not** be marked.*
- *All necessary working must be shown clearly.*
- *Mathematical tables with a list of formulae and squared papers are provided.*
- *In numerical work, take g to be 9.8 ms^{-2}.*

Include the allocation table on your answer sheet

Question	Marks
Section A	
9	
10	
11	
12	
13	
14	
15	
16	
Total	

SECTION: A (40 MARKS)

1. 40 percent of the students in certain school own personal computers. If 24 students are selected at random, find the probability that between 8 and 15 students own personal computers. (5 marks)

2. Forces of (a**i** + b**j**)N and (6**i** – 4**j**)N act at the points having position vectors (-2**i** -2**j**) m and (3**i** –**j**) m respectively. If the forces reduce to a couple, find;

(a) a and b

(b) the moment of the couple (5 marks)

3. Use the trapezium rule with 5 strips to find an approximate value for

$$\int_1^2 xe^{-2x}\,dx$$ Correct to 3 significant figures. (5 marks)

4. A random sample of 100 Adults is taken from a population. The sample is found to have a mean weight of 76.0 and variance of 144.00. Find the 97% confidence limits for the mean of the population (5 marks)
5. Two particles are travelling along a straight line AB of length 20m. At the same instant one particle starts from rest at A and travels towards B with a constant acceleration of $2ms^{-2}$ and the other particle starts from rest at B and travels towards A with a constant acceleration of $5m^{-2}$. Find how far from A the particles collide. (5 marks)
6. An examination question has two parts A and B. The probability of a student getting part A correct is $\frac{2}{3}$. If she gets part A correct, the probability that she gets part B correct is $\frac{3}{4}$, otherwise its $\frac{1}{6}$. There are three marks for a correct solution of part A, two marks for part B and a bonus mark if both parts are correct. Calculate the expected student's total mark.

(5 marks)

7. A light inextensible string of length 40 cm has its upper end fixed at a point A, and carries a mass of 2 kg at its lower end. A horizontal force applied to the mass keeps it in equilibrium, 20 cm from the vertical through A. Find the magnitude of this horizontal force and the tension in the string. (5 marks)
8. The table below shows the values of a function f(x)

X	1.0	1.1	1.2
f(x)	0.000	0.095	0.182

Use linear interpolation/extrapolation to find;

(i) f(1.16)

(ii) f (x) = 0.24 (5 marks)

SECTION B (60 MARKS)

9. (a) Show that there is real root of the equation $e^x - 2x - 5 = 0$ between -3 and -2. (3 marks)

(b) Derive the Newton-Raphson formula, to find the root of the equation in (a) above,

(3 marks)

Hence construct a flow chart that

(i) reads the first approximation X_0,

(ii) computes the root to 3 decimal places,

(iii) prints the root. (4 marks)

(c) Perform a dry run for your flow chart using $x_0 = -2.5$ (2 marks)

10. The height of pupils in a school is normally distributed with mean 130 cm and standard deviation of 7 cm. If a pupil is selected at random. Find the probability that his/her height is

(i) above 144 cm (2 marks)

(ii) less than 123 cm (2 marks)

(b) determine the lower and upper quartile heights (4 marks)

(c) If 4000 pupils are selected at random, determine the number whose height will lie between 126.5 cm and 137 cm. (4 marks)

11. A car of mass 750 kg has a maximum power of 30 kW and moves against a constant resistance to motion of 800 N. Find;

(a) the maximum speed of the car;

(i) on the level road. (3 marks)

(ii) Up an incline of slope 1 in 10 (4 marks)

(b) the acceleration down the same incline if it moves at 40 ms^{-1} (5 marks)

12 The table below shows the time taken to load vehicles in a day

Time (minutes)	Number of vehicles
40 - 45	4
45 - 50	13
50 - 55	17
55 - 60	44
60 - 70	59
70 - 80	7

(a) Draw an histogram and use it to estimate the modal time. (5 marks)

(b) Find the: (i) Mean time, (ii) Median time (iii) standard deviation of the time (7 marks)

13. A particle is projected from a point which is 2 m above the ground level with a velocity of 40 ms^{-1} at an angle of 45^0 to the horizontal. Find its horizontal distance from the point of projection where it hits the ground. (6 marks)

(b) A particle is projected inside a tunnel which is 2 m high. If the initial speed is u show that the maximum range inside the tunnel is $4\sqrt{\left(\frac{u^2-4g}{g}\right)}$ (6 marks)

14. The numbers y and z are approximated with possible errors of Δy *and* Δz respectively.

(a) Show that the maximum relative error in the quotient $\frac{y}{z}$ is given by $\left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right|$ (5 marks)

(b) Given that x = 20.136, y = 15.3 and z = 9.5342 are rounded to the given number of decimal places, find the limits within which the exact value of $\frac{y-x}{y+z}$ is expected to lie (7 marks)

15. A continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} 0.5k\,x^2 & 0 \le x < 2 \\ k(4-x) & 2 < x \le 4 \\ 0 & otherwise \end{cases}$$

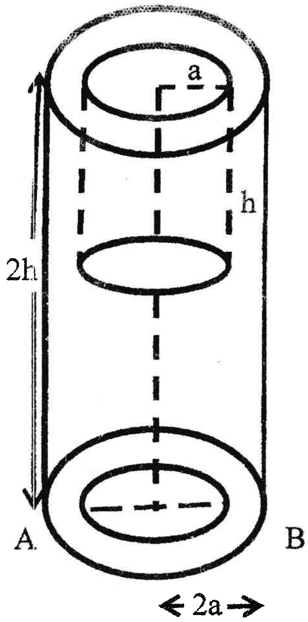
(a) Sketch the graph of $f(x)$ (3 marks)

(b) Determine the value of k (3 marks)

(c) Find the (i) Mean (ii)Median (6 marks)

16. (a)The rectangle ABCD has AB = 4 cm and AD = 2 cm. Particles of mass 3 kg, 5 kg, 1 kg and 7 kg are placed at the points A, B, C and D respectively. Find the distance of the centre of gravity of the system from each of lines AB and AD. (6 marks)

(b) The diagram below shows a uniform cylinder of radius 2a and height 2h with a cylindrical hole of radius **a** and height **h** drilled centrally at one plane end.



Show that the centre of gravity of the remaining solid is $\frac{13}{14}h$ from the base AB. (6 marks)

END

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MOCKS P 425/2
MATHEMATICS. UACE.

Q.1

$n = 24,\ p = 0.4,\ q = 0.6$

$P(8 < x < 15) = ?$

$\mu = np$
$\mu = 24 \times 0.4 = 9.6$

$\sigma^2 = npq$
$= 9.6 \times 0.6$
$\sigma^2 = 5.76$
$\sigma = \sqrt{5.76}$
$\sigma = 2.4$ B1

$P(8 < x < 15) = P(9 \le x \le 14)$

$= P(8.5 \le x < 14.5)$.

Normalizing

$= P($

$= P\left(\frac{8.5 - 9.6}{2.4} \le z < \frac{14.5 - 9.6}{2.4}\right)$ M1 M1

$= P(-0.458 \le z < 2.042)$. B1

$= 0.1765 + 0.4794$
$= 0.6559$. A1

05

Q. 2

(a)

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ M1

$a+6=0$

and $b-4=0$

$\therefore$ $a=-6$ and $b=4$. A1 A1

b) $O\circlearrowright$ $G = \begin{vmatrix} -2 & -6 \\ -2 & 4 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ -1 & -4 \end{vmatrix}$ M1

$= -8+12 + -12 - -6$

$= -26\,Nm$

$= 26\,Nm$ clockwise A1

5

Q.3 $h = \frac{2-1}{5} = 0.2$

x	y_0, y_5	y_1, y_2, y_3, y_4
1.0	0.13534	
1.2	~~0.10886~~	0.10886
1.4		0.08513
1.6		0.06522
1.8		0.04918
2.0	0.03663	
	0.17197	0.30839

B1 B1 B1

$$\int_1^2 x e^{-2x} dx \approxeq$$

$$\approxeq \frac{1}{2} \times 0.2 \left(0.17197 + 2 \times 0.30839\right)$$ M1

$$\approxeq 0.1 (0.78875$$

$$\approxeq 0.078875$$

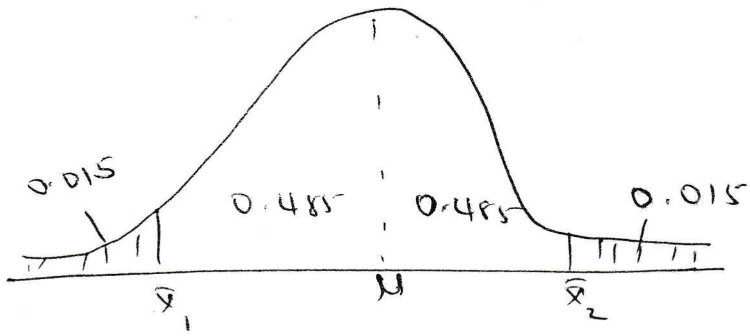
$$\approxeq 0.0789 \quad 3 \text{ s.f.}$$ A1

05

3

Q4 $n = 100$, $\bar{x} = 76.0$, $s^2 = 144.00$

Find 97% Confidence limits.



P	Q	Z
0.485	0.015	2.17

B1

$$\Rightarrow 2.17 = \frac{\bar{x}_2 - 76.0}{\frac{12}{\sqrt{100}}}$$ M1

$$\therefore \bar{x}_2 = 76 + 2.17 \times \frac{12}{10}$$

$$\bar{x}_2 = 76 + 2.17 \times 1.2$$

$$\bar{x}_2 = 78.604$$

Upper limit = 78.604 A1

From $-2.17 = \frac{\bar{x}_1 - 76.0}{1.2}$ M1

$$\bar{x}_1 = 76 - 2.17 \times 1.2$$

$$= 73.396.$$

Lower limit = 73.396 A1

05

Q.5

$$S = ut + \frac{1}{2}at^2$$

A → $x = \frac{1}{2}(2)t^2 \quad \ldots (i)$ B1

B: $20 - x = \frac{1}{2}(5)t^2 \quad \ldots (ii)$ B1

$$20 - x = \frac{5}{2}x$$ M1

$$20 - x = \frac{5}{2}x$$ B1

$$20 = \frac{7}{2}x$$

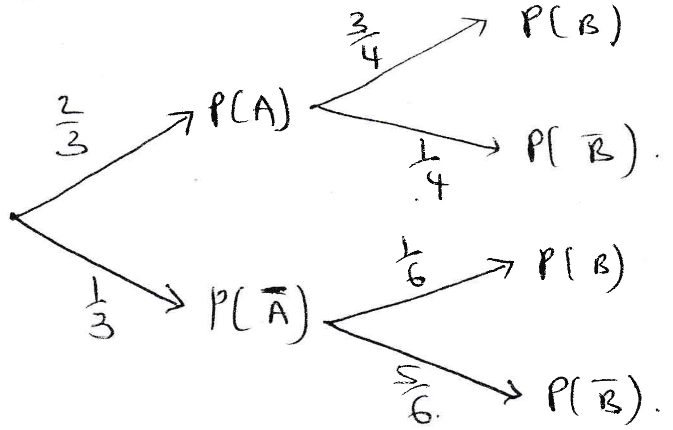
$$x = \frac{40}{7}\text{ m}$$ A1

or $x = 5.7143$ m.

05

5

Q. 6. A is for correct part of A
B " for correct part of B



both correct =

$P(A \cap B) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

$P(A \cap \bar{B}) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$ A correct

$P(\bar{A} \cap B) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$ B correct B2

$P(\bar{A} \cap \bar{B}) = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$ failed both.

Let x be mark scored.

x	$P(X=x)$	$xP(X=x)$
0	$\frac{5}{18}$	0
2	$\frac{1}{18}$	$\frac{2}{18}$
3	$\frac{1}{6}$	$\frac{3}{6}$
6	$\frac{1}{2}$	3

B1 M1

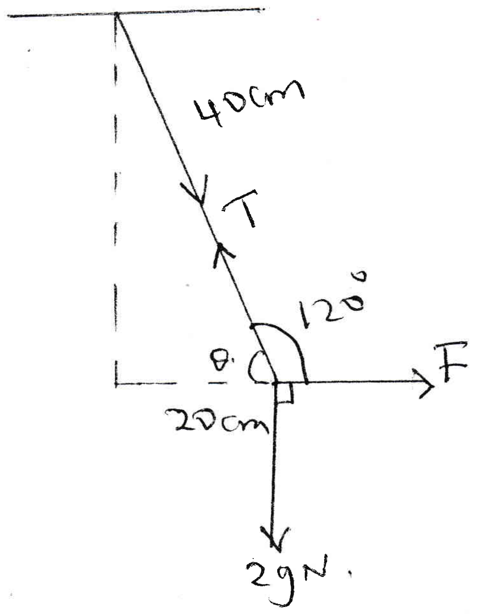
$E(X) = 3.61$.

Expected total mark = 3.61 A1

(05)

6

Q. 7



B1

$$\cos\theta = \frac{20}{40} = 0.5$$

$$\theta = 60^\circ.$$

$$\frac{T}{\sin 90^\circ} = \frac{F}{\sin 150^\circ} = \frac{2g}{\sin 120^\circ}$$ M1

$$T = \frac{2\times 9.8}{\sin 120^\circ} = \underline{22.6321N.}$$ A1

$$F = \frac{2\times 9.8 \sin 150^\circ}{\sin 120^\circ}$$ M1

$$= 11.3161N.$$ A1

5

7

Q.8

a.

x	1.1	1.16	1.2
f(x)	0.095	y	0.182

B1 (mobile)

$$\frac{y-0.095}{1.16-1.1} = \frac{0.182-0.095}{1.2-1.1}$$ M1

$y-0.095 = 0.87 \times 0.06 = 0.0522$

~~$y = 0.965$~~

$y = 0.1472$ A1

b.

x	1.1	1.2	x_1
f(x)	0.095	0.182	0.24

$$\frac{x_1-1.2}{0.24-0.182} = \frac{1.2-1.1}{0.182-0.095}$$ M1

$x_1-1.2 = 0.066667$

$x_1 = 1.267$. A1

$\boxed{05}$

Q. 1

a)

$f(x) = e^{x} - 2x - 5 = 0$

$f(-3) = e^{-3} + 6 - 5 = 1.0498$ B1

$f(-2) = e^{-2} + 4 - 5 = -0.8647$ B1

Since $f(-3) > 0$ and $f(-2) < 0$

Implies there is a root

between -3 and -2.

OR $f(-3)\, f(-2) < 0$.

Implies there is a root between -3 and -2. A1

b)

$f(x) = e^{x} - 2x - 5$

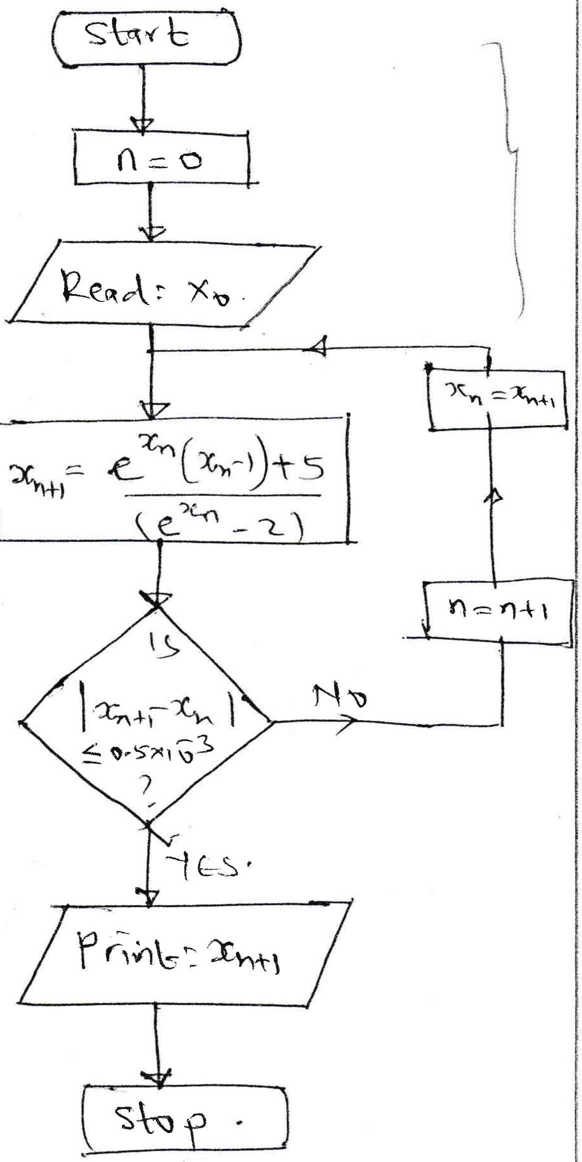
$f'(x) = e^{x} - 2$ B1

$x_{n+1} = x_n - \dfrac{(e^{x_n} - 2x_n - 5)}{(e^{x_n} - 2)}$ M1

$= \dfrac{x_n e^{x_n} - 2x_n - e^{x_n} + 2x_n + 5}{e^{x_n} - 2}$

$= \dfrac{e^{x_n}(x_n - 1) + 5}{(e^{x_n} - 2)}$ B1

Q.9 (b)



B1

B1

B1

B1

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	-2.5	-2.4572	0.0428
1	-2.4572	-2.4572	0.0000

B1

$\therefore$ root $= -2.457$ A1

112

10

Q.10 $\mu = 130$ cm , $\sigma = 7$ cm.

$P(x > 144) =$

$P\left(z > \frac{144-130}{7}\right)$ M1

$P(z > 2) = 0.0228.$

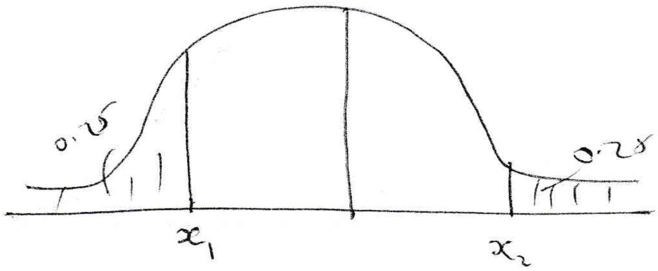
$\underline{P(x > 144) = 0.0228.}$ A1

(ii) $P(x < 123) =$

$P\left(z < \frac{123-130}{7}\right)$ M1

$P(z < -1)$

$\underline{= P(z < -1) = 0.1587}$ A1



p	Q	z
0.25	0.25	0.674

$\Rightarrow 0.674 = \frac{x_2 - 130}{7}$ B1 either

$\Rightarrow 0.674 = \frac{x_2 - 130}{7}$ B1 either

Upper Quartile height = $\underline{134.718}$.

then $-0.674 = \frac{x_1 - 130}{7}$ B1

$x_1 = 130 - 0.674 \times 7$ M1 upper

$x_1 = 125.282$ A1 ~~[illegible]~~ lower //

Lower Quartile = $\underline{125.282}$

Q. 10

(c) $P(126.5 < X < 137)$

$= P\left(\frac{126.5 - 130}{7} < Z < \frac{137 - 130}{7}\right)$ M1

$= P(-0.5 < Z < 1)$.

$= 0.1915 + 0.3413$ B1

$= 0.5328$

Number of pupils = M1

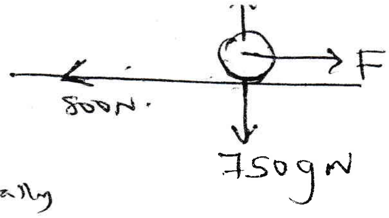
0.5328×4000.

$= 2131$ pupils. A1

$\boxed{12}$

12

Q.11



Horizontally

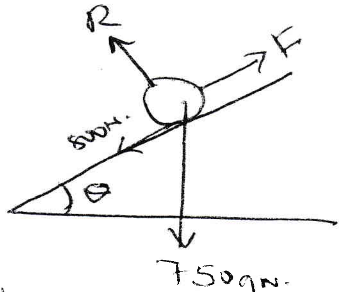
$F - 800 = 0$

$F = 800.$ B1

$F = \frac{P}{V} = \frac{30,000}{V_{max}} = 800$ M1

$V_{max} = 37.5 \, ms^{-1}.$ A1

ii.



$\sin\theta = \frac{1}{10}.$ B1

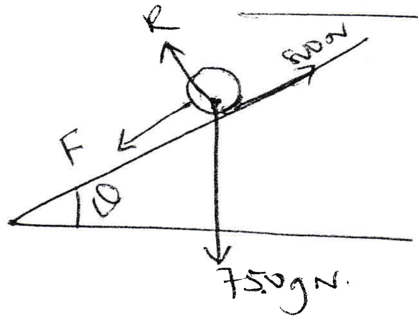
Resolving ∥ to plane

$F - 800 - 750g\sin\theta = 0.$ B1

$\frac{30,000}{V_{max}} = 800 + 75 \times 9.8$ M1

$V_{max} = 19.544 \, ms^{-1}.$ A1

b.



B1
B1

$F + 750g\sin\theta - 800 = 750a.$ M1

$\frac{30,000}{400} + 75g - 800 = 750a.$ M1

$a = \frac{1}{75} ms^{-2} = 0.0133 ms^{-2}.$ A1

12

13

Q.12

x	f	c	f.d	fx	fx²	cf
42.5	4	5	0.8	170	7225	4
47.5	13	5	2.6	617.5	29331.25	17
52.5	17	5	3.4	892.5	46856.25	34
57.5	44	5	8.8	2530	145475	78 ←
65	59	10	5.9	3835	249275	137
75	7	10	0.7	525	39375	144

$\Sigma f = 144$

$\Sigma fx = 8,570$ $\quad \Sigma fx^2 =$ ~~386,610.25~~ $= 517537.5$

B1 B1

i, mean time $= \dfrac{8570}{144}$ M1

$= 59.514$ minutes (with or out units) A1

median time $= 55 + \left(\dfrac{72-34}{44}\right) \times 5.$

$= 59.318$ A1

iii, $S.D = \sqrt{\dfrac{\cancel{386610.25}}{144} - \left(\dfrac{8570}{144}\right)^2}$

$\pm$ $S.D = \sqrt{\dfrac{517537.5}{144} - \left(\dfrac{8570}{144}\right)^2}$ M1

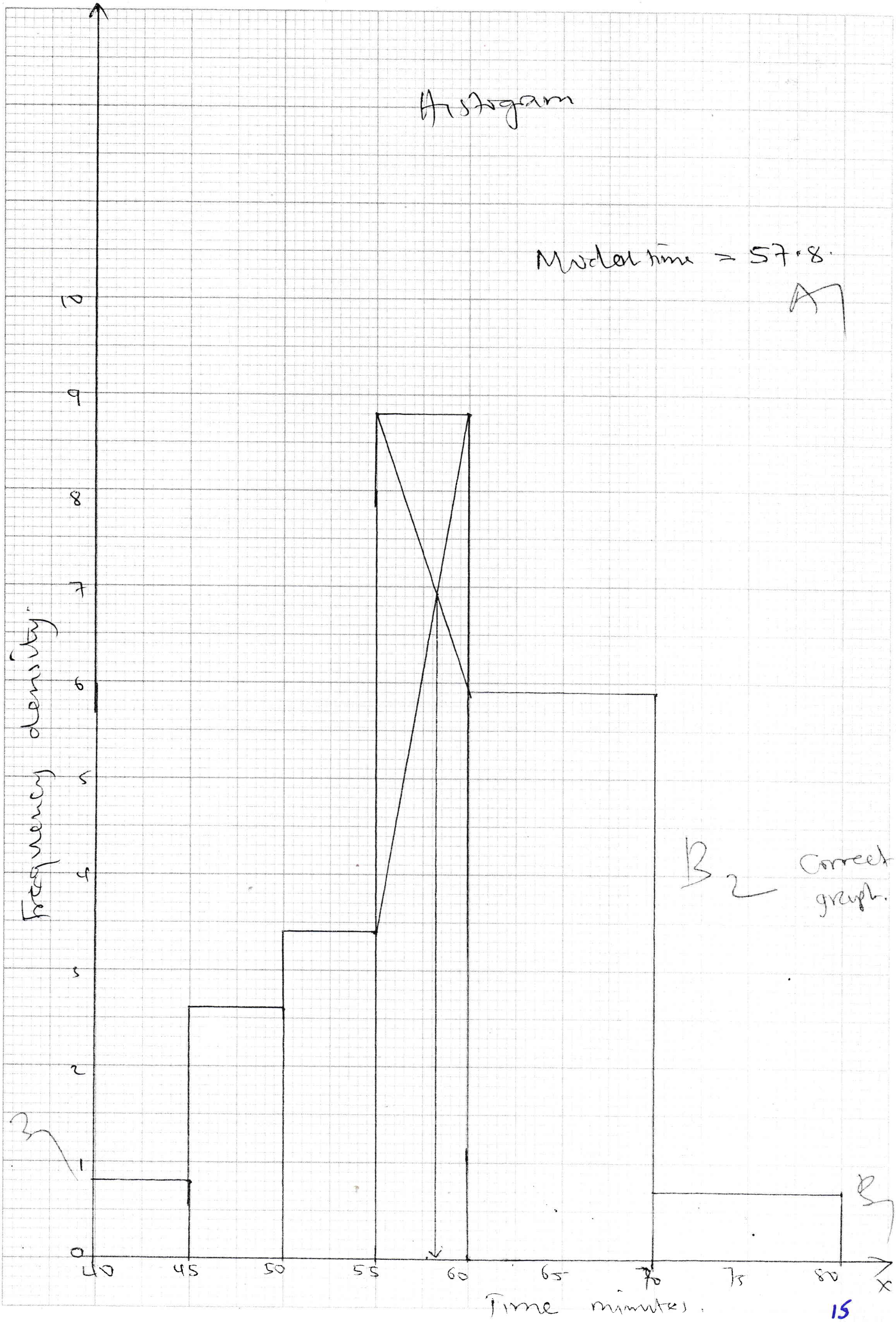
$S.D = \sqrt{3594.01042 - 3541.90297}$

$S.D = \sqrt{52.1074461}$

$S.D = 7.219$ minutes. A1

14

Histogram



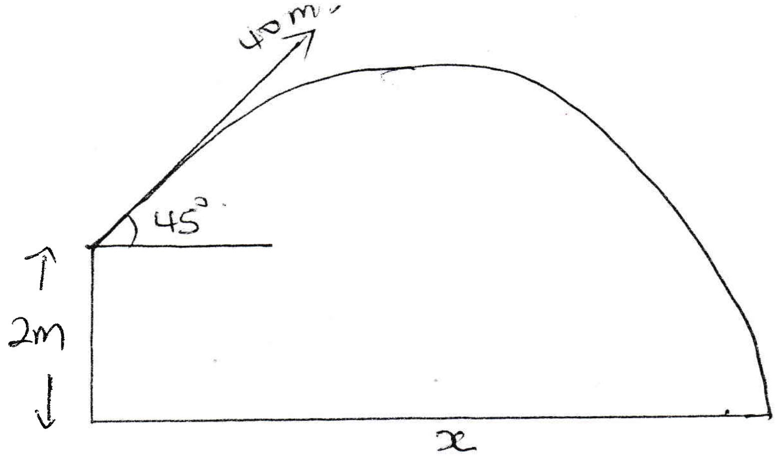
Modal time = 57.8.

A1

B2 Correct graph.

B1

Q.13



B1

$x = 40\cos 45° t$

$-2 = 40\sin 45° t - \frac{1}{2}gt^2$ M1

$4.9t^2 - 40\sin 45° t - 2 = 0.$ M1

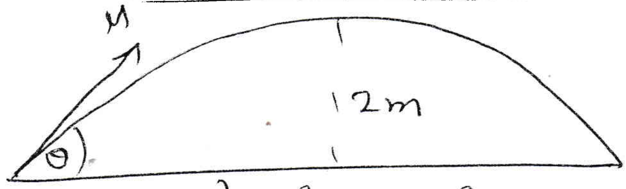
$t = \dfrac{40\sin 45° \pm \sqrt{(-40\sin 45°)^2 - 4\times 4.9\times(-2)}}{2\times 4.9}$ B1

$t = 5.822\text{ s}$ or $-0.06986\text{ s}.$ M1

$\therefore\ x = 40\cos 45° \times 5.822$

$= \underline{165.2424\text{ m}}$ A1

b.



B1

$2 = \dfrac{u^2\sin^2\theta}{2g}$, $\sin^2\theta = \dfrac{4g}{u^2}$

$R = \dfrac{2u^2\cos\theta\sin\theta}{g}$ M1

$R = \dfrac{2u^2 \cdot \sqrt{1-\sin^2\theta} \cdot \sin\theta}{g}$ M1

$= \dfrac{2u^2}{g} \cdot \sqrt{\left(1 - \dfrac{4g}{u^2}\right)} \cdot \dfrac{2}{u}\sqrt{g}$ M1

$= 4u \cdot \dfrac{\sqrt{u^2-4g}}{\cancel{u}} \cdot \dfrac{\sqrt{g}}{g}$ B1

$R = 4\sqrt{(u^2-4g)/g}$ B1

12

16

Q. 14

$$\text{Error} = \left(\frac{y+\Delta y}{z+\Delta z}\right) - \frac{y}{z}$$

1M

$$= \frac{z(y+\Delta y) - y(z+\Delta z)}{z(z+\Delta z)}$$

$$= \frac{zy + z\Delta y - yz - y\Delta z}{z(z+\Delta z)}$$

$$= \frac{z\Delta y - y\Delta z}{z^2\left(1+\frac{\Delta z}{z}\right)}$$

3M

If $\Delta z << z$ then $\frac{\Delta z}{z} \simeq 0$.

$$\text{Error} = \frac{z\Delta y - y\Delta z}{z^2}$$

$$R.E = \left|\left(\frac{z\Delta y - y\Delta z}{z^2}\right) \div \frac{y}{z}\right|$$

1M

$$= \left|\frac{(z\Delta y - y\Delta z) \cdot z}{z^2 \cdot y}\right|$$

$$= \left|\frac{\Delta y}{y} - \frac{\Delta z}{z}\right|$$

3M

$$\text{Max } R.E \leq \left|\frac{\Delta y}{y}\right| + \left|-\frac{\Delta z}{z}\right|$$

$$= \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right|.$$

3M

Q.14 (b)

$x = 20.136,\ y = 15.3,\ z = 9.5342$

Upper limit

$$= \frac{15.35 - 20.1355}{15.35 + 9.53425}$$

$$= -0.19231$$

lower limit $= \dfrac{15.25 - 20.1365}{15.25 + 9.53415}$

$$= -0.19716.$$

$\therefore$ lower limit $= -0.19716$

Upper limit $= -0.19231$

B1 M1

M1 3

3

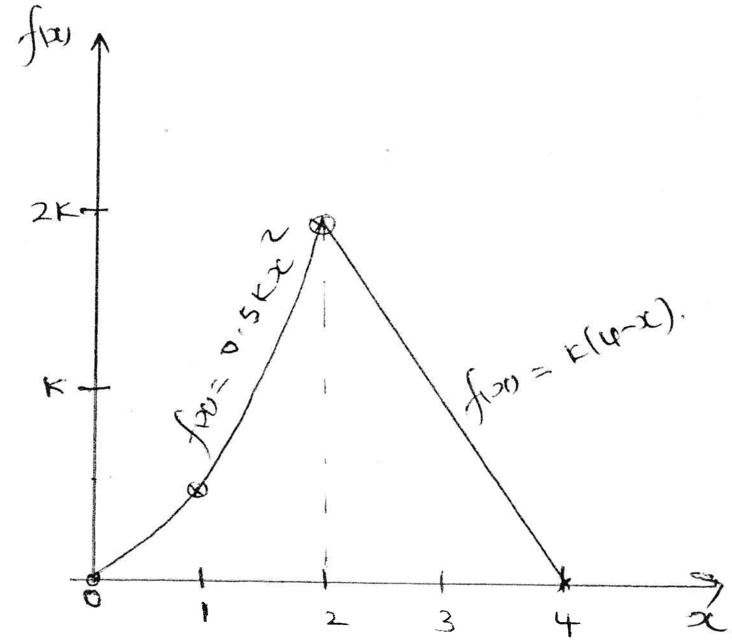
A1 A1

12

Q.15

x	0	1	2	4
$f(x)$	0	$0.5K$	$2K$	0

a)



B1 B1

B1

b)

$$\int_0^2 \frac{1}{2}Kx^2\,dx + \int_2^4 K(4-x)\,dx = 1$$

$$\frac{K}{2}\left[\frac{x^3}{3}\right]_0^2 + K\left[4x - \frac{x^2}{2}\right]_2^4 = 1$$ M1

$$\frac{K}{2}\left(\frac{8}{3}\right) + K\left(8-6\right) = 1$$ B1

$$\frac{4K}{3} + 2K = 1$$

$$10K = 3$$

$$K = 0.3 = \frac{3}{10}$$ A1

Mean $E(X)$ =

$$\int_0^2 x \cdot \frac{K}{2}x^2\,dx + \int_2^4 Kx(4-x)\,dx$$

$$= \frac{K}{2}\int_0^2 x^3\,dx + K\int_2^4 (4x - x^2)\,dx$$

$$= \frac{K}{2}\left[\frac{x^4}{4}\right]_0^2 + K\left[2x^2 - \frac{x^3}{3}\right]_2^4$$ M1

$E(X) = 0.6 + 0.3\left(24 - \frac{56}{3}\right)$ B1

$E(X) = 2.2$ A1

Median:

$\int_2^4 k(4-x)\,dx = \text{Area}$

$= \frac{1}{2} \times 2 \times 0.6 = 0.6.$

Hence median lies between 2 and 4.

$\int_m^4 k(4-x)\,dx = 0.5$

$0.3\left[4x - \frac{x^2}{2}\right]_m^4 = 0.5$ M1

$0.3\left(8 - 4m + \frac{m^2}{2}\right) = 0.5$

$0.3\left(\frac{16 - 8m + m^2}{2}\right) = 0.5$

$3(16 - 8m + m^2) = 10.$

$3m^2 - 24m + 38 = 0$ B1

$m = \frac{24 \pm \sqrt{576 - 4 \times 3 \times 38}}{6}$

$m = 4 \pm 1.826$

median $= 2.174$ A1

Since 5.826 is outside the range.

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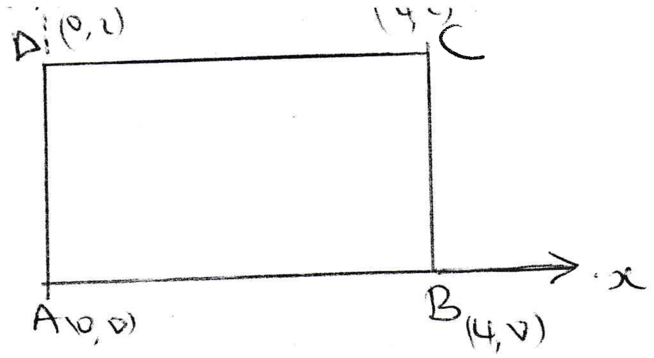
Q: 16.



Total weight $= 3g + 5g + 1g + 7g = 16g\,N$. B1

Taking moment about AB and AD.

$$16g\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 3g\begin{pmatrix}0\\0\end{pmatrix} + 5g(4,0) + g\begin{pmatrix}4\\2\end{pmatrix} + 7g\begin{pmatrix}0\\2\end{pmatrix}$$ M1 M1

$16\bar{x} = 20 + 4$ B1

$\bar{x} = 1.5$.

$16\bar{y} = 0 + 0 + 2 + 14$.

$\bar{y} = \frac{16}{16} = 1$. B1

Centre of gravity is 1.5cm from AD and 1cm from AB. A1

b.

Portion		C.O.G. from AB.
Cylinder	$8\pi a^2 h \rho$	h
Hole	$\pi a^2 h \rho$	$\frac{3}{2}h$.
Remainder	$7\pi a^2 h \rho$	$\bar{y}$.

B1 B1 B1

Taking moments about AB.

$$8\pi a^2 h \rho h = \pi a^2 h \rho \times \frac{3}{2}h + 7\pi a^2 h \rho.\ \bar{y}.$$ M1 M1

$8h = \frac{3}{2}h + 7\bar{y}$.

$8h - \frac{3}{2}h = 7\bar{y}$.

$\bar{y} = \frac{13}{14}h$. A1

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